Math Virtual Learning

Precalculus with Trigonometry

Students will find the exact value of angle measures for tangent using Sum and Difference Identities. Students will also use the Sum and Difference Formulas to identify the sum or difference of angles when given the ratio for sine, cosine, or tangent.

April 29, 2020

Precalculus with Trigonometry
Lesson: April 29th, 2020

Objective/Learning Target:
Students will find the exact value of angle measures for tangent using Sum and Difference Identities. Students will also use the Sum and Difference Formulas to identify the sum or difference of angles when given the ratio for sine, cosine, or tangent.

Objective 1 for today's lesson: Finding the exact value of angle measures for tangent using Sum and Difference Identities.

Let's Get Started:

Watch Video:
[Sum and Difference for Tangent](#)

Sum and Difference Identities

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Example

*Note the step in red…

In order to simplify the ratio, you must multiply by the conjugate of the expression

USE SUM FORMULA TO FIND EXACT VALUE

$$\tan \frac{13\pi}{12} = \tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$

$$\frac{\tan \frac{\pi}{4} + \tan \frac{\pi}{3}}{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{3}} = \frac{-1 + \sqrt{3}}{1 - (-1)\left(\sqrt{3}\right)}$$

$$\frac{-1 + \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{-4 - 2\sqrt{3}}{1 - 3}$$

$$\frac{-4 - 2\sqrt{3}}{-2} = 2 + \sqrt{3}$$

Objective 2 of Today's Lesson

Use the Sum and Difference Formulas to identify the sum or difference of angles when given the ratio for sine, cosine, or tangent.
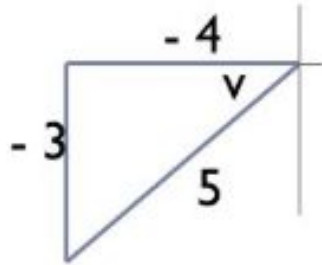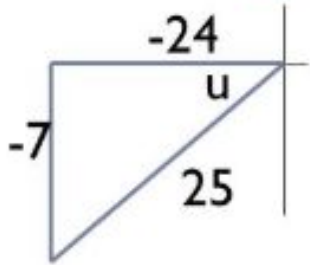
Watch Video: Find Sum and Difference given ratios

Example

Then plug into the difference formula for cosine:

Find the exact value of the Cos (u − v) using the given information:

$cos(u - v) = cosucosv + sinusinv$

$Sin\ u = -\dfrac{7}{25}$ $Cos\ v = -\dfrac{4}{5}$ Both u and v and in quadrant III

$= \dfrac{-24}{25} \cdot \dfrac{-4}{5} + \dfrac{-7}{25} \cdot \dfrac{-3}{5}$

When you are given 2 different criteria, you must draw 2 different triangles

$= \dfrac{96}{125} + \dfrac{21}{125} = \dfrac{117}{125}$

Practice

Use the Sum and Difference Identities to solve the following:

1. Find the exact value of $\tan 285°$

2. Find the exact value of $\tan 75°$.

3. Find the exact value of $\sin(u + v)$ given

$$\sin u = \frac{4}{5}, \text{ where } 0 < u < \frac{\pi}{2}, \text{ and } \cos v = -\frac{12}{13}, \text{ where } \frac{\pi}{2} < v < \pi.$$

4. Given same ratios above, find $\cos(u - v)$

Practice - **ANSWERS**

$$\tan(330° - 45°) = \frac{\tan 330° - \tan 45°}{1 + \tan 330° \tan 45°}$$

$$= \frac{-\frac{\sqrt{3}}{3} - 1}{1 - \frac{\sqrt{3}}{3} \cdot 1} = \frac{-3 - \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{-3 - \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{-9 - 6\sqrt{3} - 3}{9 - 3}$$

$$= \frac{-12 - 6\sqrt{3}}{6}$$

$$= -2 - \sqrt{3}$$

$$\tan 75° = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

$$\tan 75° = \frac{1 + \frac{1}{\sqrt{3}}}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$\tan 75° = \frac{\left(\frac{\sqrt{3} + 1}{\sqrt{3}}\right)(\sqrt{3})}{\left(\frac{\sqrt{3} - 1}{\sqrt{3}}\right)(\sqrt{3})}$$

$$\tan 75° = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\tan 75° = \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1}\right)\left(\frac{\sqrt{3} + 1}{\sqrt{3} + 1}\right)$$

$$\tan 75° = \frac{3 + 2\sqrt{3} + 1}{3 - 1}$$

$$\tan 75° = 2 + \sqrt{3}$$

Practice - ANSWERS

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$$= \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$$

$$= -\frac{48}{65} + \frac{15}{65}$$

$$= -\frac{33}{65}$$

$\cos(u - v) = \cos u \cos v + \sin u \sin v$

$$= \frac{3}{5} \cdot \frac{-12}{13} + \frac{3}{5} \cdot \frac{5}{13}$$

$$= \frac{-16}{65}$$

Additional Practice and Resources:

Additional ResourceVideos:
[Sum and Difference for Tangent](#)

[Sum and Difference Identities Given Ratios](#)

Additional Practice:
[Finding Exact Values - Khan Academy](#)

[Sum and Difference Identities Practice](#)
Try problems 13-16 on p.6 (Answers are on page 10)